The Steady Magnetic Field المجال المغناطيسي الثابت

7.1 BIOT-SAVART Law

The source of the steady magnetic field may be a ***permanent magnet***, an ***electric field changing linearly with time***, or a ***direct current***. We will largely ignore the permanent magnet and save the time-varying electric field for a later discussion. Our present study will concern the magnetic field produced by a differential dc element in free space

Biot-Savart's law states that **"*the magnetic field intensity* dH *produced at a point P by the differential current element I dl is proportional to the product I dl and the sine of the angle between the element and the line joining P to the element and is inversely proportional to the square of the distance R between P and the element".***

The direction of the magnetic field intensity is normal to the plane containing the differential filament and the line drawn from the filament to the point *P as shown in Figure 7.1*.

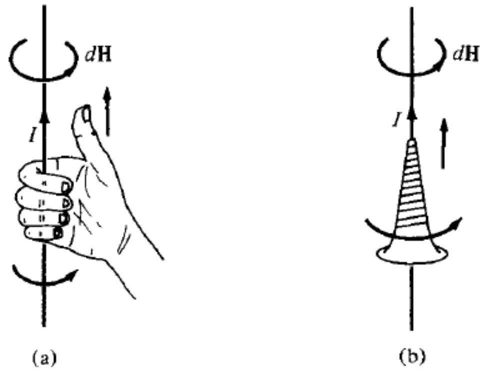
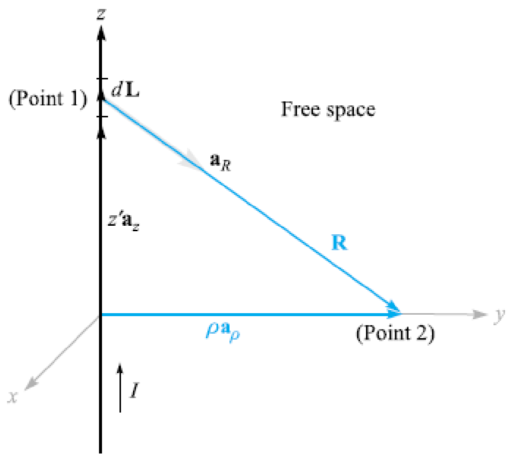


Figure 7.1 the direction of *dH* using (a) the right-hand rule, or (b) the right-handed screw rule.

We can have different current distributions: line current, surface current, and volume current. If we define K as the surface current density (in A/m) and J as the volume current density (in A/m^2),

$H = \oint \frac{I\, dL \times \mathrm{a}_R}{4\pi R^2}$	Line current
$H = \oint \frac{K\, dS \times \mathrm{a}_R}{4\pi R^2}$	Surface current
$H = \oint \frac{\mathrm{J}\, dv \times \mathrm{a}_R}{4\pi R^2}$	Volume current

Consider an ***infinitely long straight filament*** carrying a direct current I is located along z-axis



$$H = \oint \frac{I\, dL \times a_R}{4\pi R^2}$$

$$dL = dz\mathbf{a}_z$$

$$R = \rho\mathbf{a}_\rho - z\,\mathbf{a}_z$$

$$|R| = \sqrt{\rho^2 + z^2}$$

$$H = \int_{-\infty}^{\infty} \frac{I\, dz\mathbf{a}_z}{4\pi\left(\sqrt{\rho^2+z^2}\right)^2} \times \frac{\rho\mathbf{a}_\rho - z\,\mathbf{a}_z}{\sqrt{\rho^2+z^2}}$$

$$H = \frac{I}{4\pi}\int_{-\infty}^{\infty} \frac{\rho\, dz}{(\rho^2+z^2)^{\frac{3}{2}}}\,\mathbf{a}_\emptyset \qquad , \quad (\mathbf{a}_z \times \mathbf{a}_\rho = \mathbf{a}_\emptyset) \quad , \qquad (\mathbf{a}_z \times \mathbf{a}_z = 0)$$

$$Let\ z = \rho\ \tan u\ , \qquad dz = \rho \sec^2 u\ du$$

$$u = \tan^{-1}\frac{z}{\rho} \qquad u_1 = \tan^{-1}\frac{-\infty}{\rho} = \frac{-\pi}{2}\,, \qquad u_2 = \tan^{-1}\frac{\infty}{\rho} = \frac{\pi}{2}$$

$$H = \frac{I}{4\pi}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\rho\,\rho \sec^2 u\ du}{(\rho^2+\rho^2\tan^2 u)^{\frac{3}{2}}}\,\mathbf{a}_\emptyset$$

$$H = \frac{I}{4\pi}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\rho\,\rho \sec^2 u\ du}{(\rho^2+\rho^2\tan^2 u)^{\frac{3}{2}}}\,\mathbf{a}_\emptyset$$

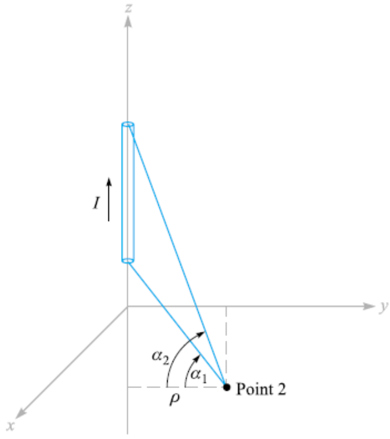
$$H = \frac{I}{4\pi\rho}[\sin u]_{\frac{-\pi}{2}}^{\frac{\pi}{2}} = \frac{I}{4\pi\rho}\left[\sin\frac{\pi}{2} - \sin\frac{-\pi}{2}\right]$$

$$H = \frac{I}{2\pi\rho}\mathbf{a}_\emptyset \qquad\qquad in\ cylindrical$$

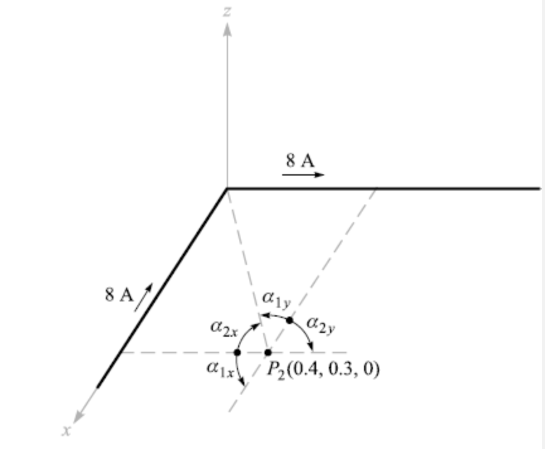
$$H = \frac{I}{2\pi\,\rho}\mathbf{a}_\emptyset = \frac{I}{2\pi}\left(\frac{-y\mathrm{a}_x + x\mathrm{a}_y}{x^2+y^2}\right) \qquad in\ cartesian\ along\ z\ axis$$

The finite-length current element is shown in Figure below. The magnetic field intensity H is most easily expressed in terms of the angles α1 and α2, as identified in the figure. The result is

$$H = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)$$



Example: Determine **H** at *P* (0.4, 0.3, 0) in the field of an 8 A filamentary current is directed inward from infinity to the origin on the positive *x* axis, and then outward to infinity along the *y* axis. As shown in Figure.



Solution:

$$H = H_1 + H_2$$

$$H_1 = \frac{I_1}{4\pi\rho_1}(\sin\alpha_2 - \sin\alpha_1)$$

$$\rho_1 = 0.3 \quad , \quad \alpha_1 = -90 \quad , \quad \alpha_2 = \tan^{-1}\frac{0.4}{0.3} = 53.1$$

$$H_1 = \frac{8}{4\pi 0.3}(\sin 53.1 + \sin 90)\mathbf{a}_\emptyset = \frac{12}{\pi}\mathbf{a}_\emptyset = \frac{-12}{\pi}\mathbf{a}_z$$

$$H_2 = \frac{I_2}{4\pi\rho_2}(\sin\alpha_2 - \sin\alpha_1)$$

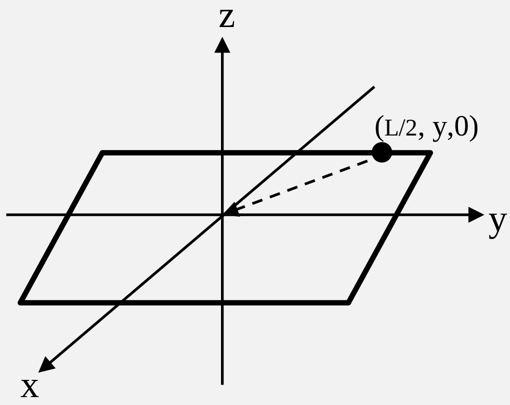
$$\rho_2 = 0.4 \quad , \quad \alpha_2 = 90 \quad , \quad \alpha_1 = -\tan^{-1}\frac{0.3}{0.4} = -36.9$$

$$H_2 = \frac{8}{4\pi 0.4}(\sin 90 + \sin 36.9)\mathbf{a}_\emptyset = \frac{-8}{\pi}\mathbf{a}_\emptyset = \frac{8}{\pi}\mathbf{a}_z$$

$$H = H_1 + H_2 = \frac{-12}{\pi}\mathbf{a}_z + \frac{8}{\pi}\mathbf{a}_z = \frac{-20}{\pi}\mathbf{a}_z$$

Example: Find H at the center of a square current loop of side L is located on xy-plane?

Solution:



$$H = \oint \frac{I\, dL \times \mathrm{a}_R}{4\pi R^2}$$

$$dL = dy\, \mathbf{a}_y$$

$$R = \frac{-L}{2}\mathbf{a}_x - y\, \mathbf{a}_y$$

$$|R| = \sqrt{\left(\frac{L}{2}\right)^2 + y^2}$$

$$H_1 = \int \frac{I\, dy\, \mathbf{a}_y}{4\pi\left(\sqrt{\left(\frac{L}{2}\right)^2 + y^2}\right)^2} \times \frac{\frac{-L}{2}\mathbf{a}_x - y\, \mathbf{a}_y}{\sqrt{\left(\frac{L}{2}\right)^2 + y^2}}$$

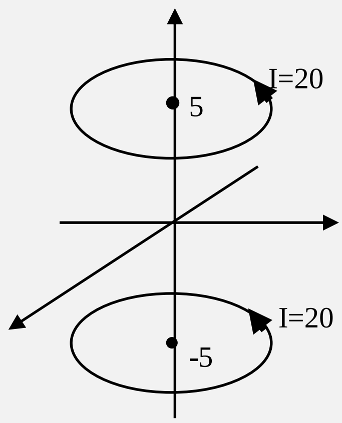
$$H_1 = \frac{I}{4\pi}\int_0^{\frac{L}{2}} \frac{\frac{L}{2}\ dy}{\left(\left(\frac{L}{2}\right)^2 + y^2\right)^{\frac{3}{2}}}\ \mathbf{a}_z \qquad , \qquad (\mathbf{a}_y \times \mathbf{a}_x = -\mathbf{a}_z) \ , \qquad (\mathbf{a}_y \times \mathbf{a}_y = 0)$$

$$H_1 = \frac{\sqrt{2}\, I}{4\pi L}\mathbf{a}_z$$

$$H = 8\, H_1 = 8 * \frac{\sqrt{2}\, I}{4\pi L}\mathbf{a}_z = \frac{2\sqrt{2}\, I}{\pi L}\mathbf{a}_z$$

Example: Two identical circular current loops of radius $\rho = 3$ and I = 20A are in parallel planes, separated on their common axis by 10 m. Find H at a point midway between the two loops?

Solution:



$$H = H_1 + H_2$$

$$H_1 = \oint \frac{I\, dL_1 \times \mathrm{a}_{R1}}{4\pi {R_1}^2}$$

$$dL_1 = \rho d\emptyset \mathbf{a}_\emptyset = 3 d\emptyset \mathbf{a}_\emptyset$$

$$R_1 = -3\mathbf{a}_\rho - 5\, \mathbf{a}_z$$

$$|R| = \sqrt{3^2 + 5^2} = \sqrt{34}$$

$$H_1 = \int_0^{2\pi} \frac{I * 3d\emptyset \mathbf{a}_\emptyset}{4\pi * 34} \times \frac{-3\mathbf{a}_\rho - 5\, \mathbf{a}_z}{\sqrt{34}} = \frac{3I}{4\pi * 34^{\frac{3}{2}}}\left[\int_0^{2\pi} 3d\emptyset \mathbf{a}_z + \int_0^{2\pi} -5d\emptyset \mathbf{a}_\rho\right] = 0.453\mathbf{a}_z$$

$$H_2 = H_1 \qquad , \qquad H = 0.908\mathbf{a}_z$$

7.2 AMPERE'S Circuital Law

Ampere's circuital law states that ***"the line integral of H about any closed path is exactly equal to the direct current enclosed by that path"***

$$\oint \mathrm{H}.\, dL = I_{enc}$$

We choose a path, to any section of which **H** is either ***perpendicular*** or ***tangential***, and along which *H* is constant. The first requirement (perpendicularity or tangency) allows us to replace the dot product of Ampere's circuital law with the product of the scalar magnitudes, except along that portion of the path where **H *is normal to the path and the dot product is zero***; the second requirement (constancy) then permits us to remove the magnetic field intensity from the integral sign. The integration required is usually trivial and consists of finding the length of that portion of the path to which **H** is parallel.

Let us again find the magnetic field intensity produced by an infinitely long filament carrying a current *I*. The filament lies on the *z* axis in free space, and the current flows in the direction given by $\mathbf{a}_z$.

the path must be a circle of radius ρ, and Ampere's circuital law becomes

$$\oint \mathrm{H}.\, dL = I_{enc}$$

$$\oint \mathrm{H}.\, dL = \int_0^{2\pi} \mathrm{H}_\emptyset\, \rho d\emptyset = \mathrm{H}_\emptyset\, \rho \int_0^{2\pi} d\emptyset = 2\pi\, \mathrm{H}_\emptyset\, \rho$$

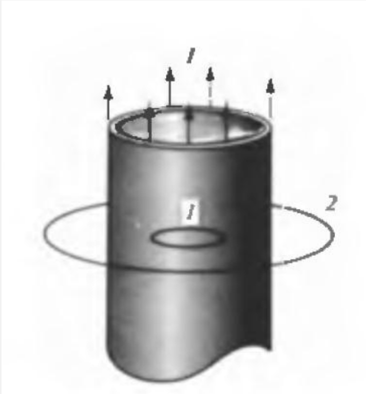
$$2\pi\, \mathrm{H}_\emptyset\, \rho = I$$

$$\mathrm{H}_\emptyset = \frac{I}{2\pi\, \rho}$$

$$\mathrm{H} = \frac{I}{2\pi\, \rho}\, \mathrm{a}_\emptyset$$

Example: A thin cylindrical conductor of radius a, infinite in length, carries a current I. Find H at all points using Ampere's law?

Solution:



For path 1 inside cylinder

$$\oint \mathrm{H}.dL = I_{enc}$$

$$I_{enc} = 0$$

$$\therefore \mathrm{H} = 0$$

For path 2 outside cylinder

$$I_{enc} = I$$

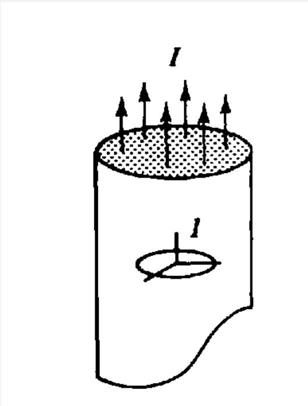
$$\oint \mathrm{H}.dL = \int_0^{2\pi} \mathrm{H}_\emptyset\, \rho d\emptyset = \mathrm{H}_\emptyset\, \rho \int_0^{2\pi} d\emptyset = 2\pi\rho\, \mathrm{H}_\emptyset$$

$$2\pi\rho\, \mathrm{H}_\emptyset = I$$

$$\mathrm{H}_\emptyset = \frac{I}{2\pi\, \rho}$$

Example: Determine H for a solid cylindrical conductor of radius a, where the current I is uniformly distributed over the cross section?

Solution:



$$for\ \rho < a$$

$$I_{enc} = I\frac{\pi\rho^2}{\pi a^2} = \frac{\rho^2}{a^2}I$$

$$\oint \mathrm{H}.dL = \int_0^{2\pi} \mathrm{H}_\emptyset\, \rho d\emptyset = \mathrm{H}_\emptyset\, \rho \int_0^{2\pi} d\emptyset = 2\pi\rho\, \mathrm{H}_\emptyset$$

$$2\pi\rho\, \mathrm{H}_\emptyset = \frac{\rho^2}{a^2}I$$

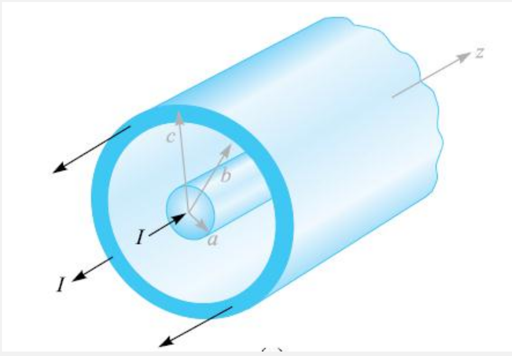
$$\mathrm{H}_\emptyset = \frac{I\rho}{2\pi a^2}$$

$for\ \rho > a$, $I_{enc} = I$, $\mathrm{H}_\emptyset = \dfrac{I}{2\pi\, \rho}$

Example: consider an infinitely long coaxial transmission line carrying a uniformly distributed total current I in the center conductor and $-I$ in the outer conductor, Find H at all points using Ampere's law?

Solution:

$for\ \rho < a$



$$I_{enc} = I\frac{\pi\rho^2}{\pi a^2} = \frac{\rho^2}{a^2}I$$

$$\oint \mathrm{H}.dL = \int_0^{2\pi} \mathrm{H}_\emptyset\ \rho d\emptyset\ = \mathrm{H}_\emptyset\, \rho \int_0^{2\pi} d\emptyset = 2\pi\rho\ \mathrm{H}_\emptyset$$

$$2\pi\rho\ \mathrm{H}_\emptyset\ = \frac{\rho^2}{a^2}I$$

$$\mathrm{H}_\emptyset\ = \frac{I\rho}{2\pi a^2}$$

$for\ a < \rho < b$

$$I_{enc} = I \qquad , \quad \mathrm{H}_\emptyset\ = \frac{I}{2\pi\ \rho}$$

$for\ b < \rho < c$

$$I_{enc} = I_1 + I_2$$

$$I_2 = -I\frac{\pi(\rho^2 - b^2)}{\pi(c^2 - b^2)}$$

$$I_{enc} = I - I\ \frac{\rho^2 - b^2}{c^2 - b^2} = \frac{c^2 - \rho^2}{c^2 - b^2}I$$

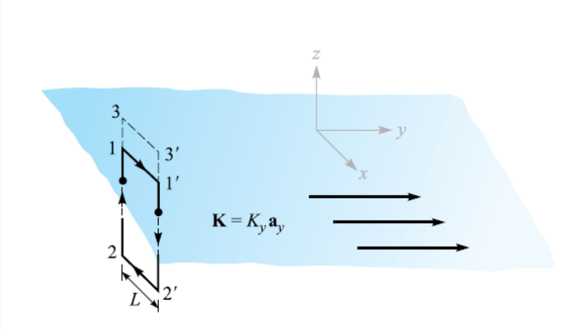
$$2\pi\rho\ \mathrm{H}_\emptyset\ = \frac{c^2 - \rho^2}{c^2 - b^2}I$$

$$\mathrm{H}_\emptyset\ = \frac{I}{2\pi\rho}\ \frac{c^2 - \rho^2}{c^2 - b^2}$$

$for\ c < \rho$

$$I_{enc} = I_1 + I_2\ \ = I - I = 0 \qquad ,\therefore\ H = 0$$

Example: consider a ***sheet of current*** flowing in the positive *y* direction and located in the *z* = 0 plane as shown in Figure below. Find H?



Solution:

$H = H_x a_x + H_y a_y + H_z a_z$

$H_y = H_z = 0$

$H = H_x a_x$

$I_{enc} = K\ L$

$\oint \mathrm{H}. dL = \int H_x a_x . dx a_x + \int H_x a_x . dz a_z = H_x L + H_x L = 2H_x L$

$\oint \mathrm{H}. dL = I_{enc}$

$2H_x L = K\ L$

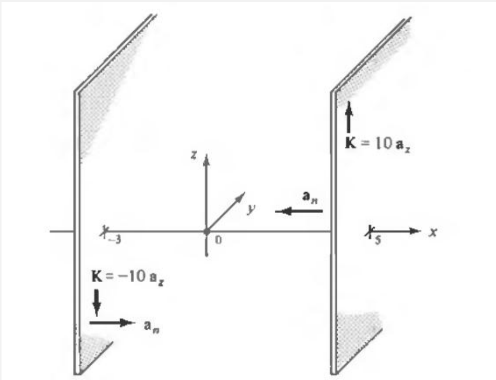
$H_x = \frac{K}{2}$

In general the H for infinite sheet current is given by:

$H = \frac{1}{2} K \times a_N$

Example: A current sheet, **K=10a$_z$** A/m, lies in the **x = 5m** plane and a second sheet, **K = -10a$_z$** A/m, is at **x = -5 m**. Find H at all points?

Solution:



$for\ -5 < x < 5$

$H_1 = \frac{1}{2} K_1 \times a_N$

$H_1 = \frac{1}{2} 10 a_z \times -a_x = -5 a_y$

$H_2 = \frac{1}{2} - 10 a_z \times a_x = -5 a_y$

$H = H_1 + H_2 = -10 a_y$

$for\ x < -5$

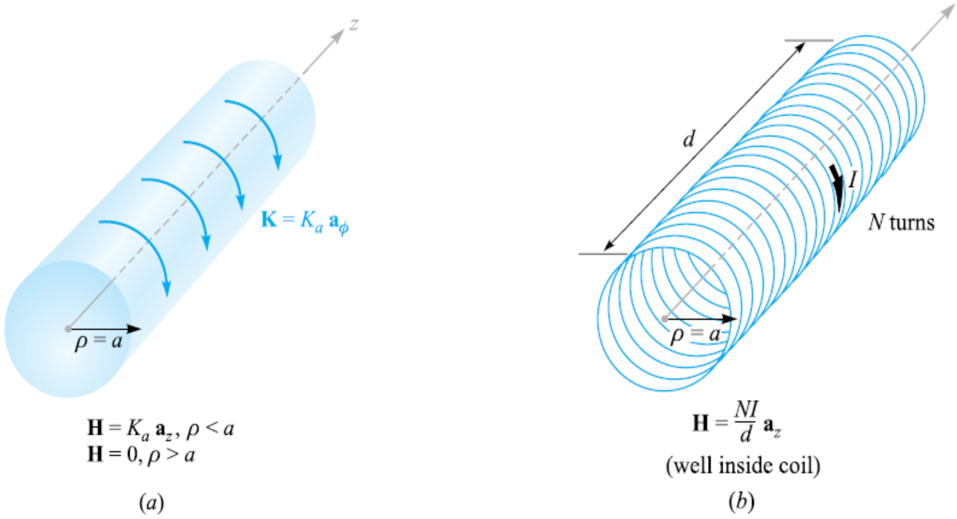
$H_1 = \frac{1}{2} 10 a_z \times -a_x = -5 a_y$

$H_2 = \frac{1}{2} - 10 a_z \times -a_x = 5 a_y$

$H = H_1 + H_2 = -5 a_y + 5 a_y = 0$

(a) An ideal solenoid of infinite length with a circular current sheet K.

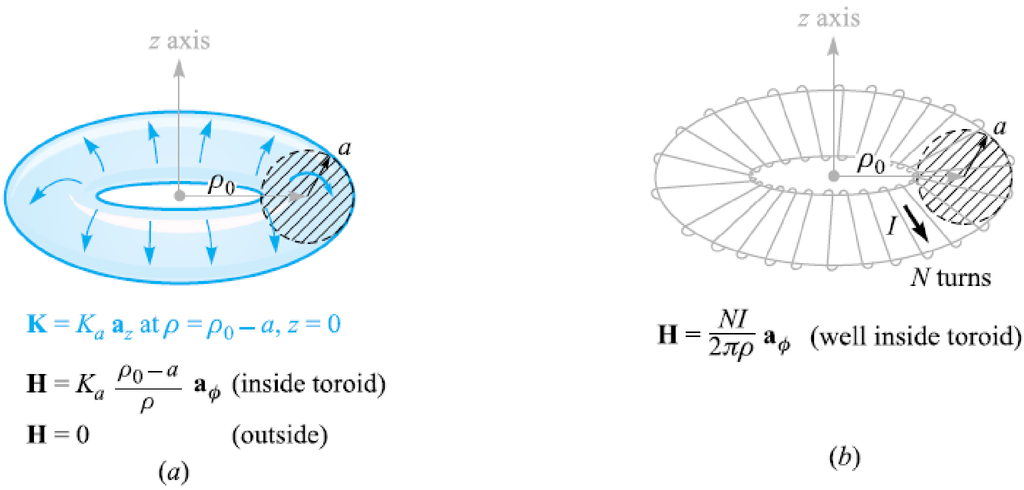
(b) An N-turn solenoid of finite length d.



$H = K_a \mathbf{a_z} \qquad \boldsymbol{\rho < a}$ $\qquad\qquad$ $H = \frac{N\,I}{d} \mathbf{a_z} \qquad \boldsymbol{\rho < a}$

$H = 0 \qquad \boldsymbol{\rho > a}$

For the toroid shown in Figure below, it can be shown that the magnetic field intensity for the ideal case



7.3 CURL

- ***In rectangular coordinates, the Curl H is given by:***

$$Curl\ H = \nabla \times \mathrm{H} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

- ***In cylindrical coordinates, the Curl H is given by:***

$$Curl\ H = \nabla \times \mathrm{H} = \begin{vmatrix} \frac{\mathbf{a}_\rho}{\rho} & \mathbf{a}_\emptyset & \frac{\mathbf{a}_z}{\rho} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \emptyset} & \frac{\partial}{\partial z} \\ H_\rho & \rho H_\emptyset & H_z \end{vmatrix}$$

$$= \left(\frac{1}{\rho}\frac{\partial H_z}{\partial \emptyset} - \frac{\partial H_\emptyset}{\partial z}\right)\mathbf{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right)\mathbf{a}_\emptyset + \left(\frac{1}{\rho}\frac{\partial(\rho H_\emptyset)}{\partial \rho} - \frac{1}{\rho}\frac{\partial H_\rho}{\partial \emptyset}\right)\mathbf{a}_z$$

- ***In spherical coordinates, the Curl H is given by:***

$$Curl\ H = \nabla \times \mathrm{H} = \begin{vmatrix} \frac{a_r}{r^2 \sin\theta} & \frac{a_\theta}{r \sin\theta} & \frac{a_\emptyset}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \emptyset} \\ H_r & rH_\theta & r \sin\theta\, H_\emptyset \end{vmatrix}$$

$$= \frac{1}{r \sin\theta}\left(\frac{\partial(\sin\theta\, \mathrm{H}_\emptyset)}{\partial \theta} - \frac{\partial H_\theta}{\partial \emptyset}\right)\mathbf{a}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial H_r}{\partial \emptyset} - \frac{\partial(rH_\emptyset)}{\partial r}\right)\mathbf{a}_\theta + \frac{1}{r}\left(\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial \theta}\right)\mathbf{a}_\emptyset$$

Frequently useful are two properties of the curl operator:

1) *The divergence of a curl is the zero scalar;* $\mathbf{.}\,(\boldsymbol{\nabla} \times \boldsymbol{A}) = \mathbf{0}$ *, for any vector field A.*
2) *The curl of a gradient is the zero vector;* $\boldsymbol{\nabla} \times (\boldsymbol{\nabla A}) = \mathbf{0}$

7.4 Relationship of J and H

If the magnetic field H is known throughout a region, then the curl H will produce the current density J for that region.

$$\boldsymbol{\nabla} \times \boldsymbol{H} = \mathbf{J}$$

This is the ***second of Maxwell's*** four equations as they apply to non-time-varying conditions.

We may also write the ***third*** of these equations at this time; it is the point form of $\oint E.dl = 0$ or

$$\boldsymbol{\nabla} \times \boldsymbol{E} = \mathbf{0}$$

Example: A long, straight conductor cross section with radius **a** has a magnetic field strength $H = \frac{I\rho}{2\pi a^2}\mathrm{a}_\emptyset$ within the conductor$(\rho < a)$ and $H = \frac{I}{2\pi\rho}\mathrm{a}_\emptyset$ for $(\rho > a)$. Find J in both regions?

Solution:

$$H = \frac{I\rho}{2\pi a^2}\mathrm{a}_\emptyset \qquad ,H_\rho = 0 \qquad ,H_\mathrm{z} = 0$$

$$\nabla \times H = \left(\frac{1}{\rho}\frac{\partial H_z}{\partial \emptyset} - \frac{\partial H_\emptyset}{\partial z}\right)\mathbf{a}_{\boldsymbol{\rho}} + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right)\mathbf{a}_\emptyset + \left(\frac{1}{\rho}\frac{\partial(\rho H_\emptyset)}{\partial \rho} - \frac{1}{\rho}\frac{\partial H_\rho}{\partial \emptyset}\right)\mathbf{a}_{\mathbf{z}}$$

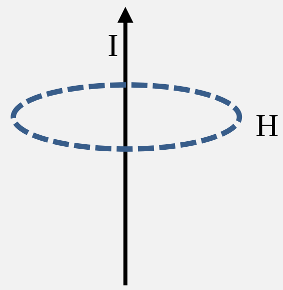
$$\nabla \times H = -\frac{\partial H_\emptyset}{\partial z}\mathbf{a}_{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial(\rho H_\emptyset)}{\partial \rho}\mathbf{a}_{\mathbf{z}}$$

$$\nabla \times H = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho\frac{I\rho}{2\pi a^2})\mathbf{a}_{\mathbf{z}}$$

$$\nabla \times H = \frac{1}{\rho}\frac{I\rho}{2\pi a^2}\mathbf{a}_{\mathbf{z}} = \frac{I}{\pi a^2}\mathbf{a}_{\mathbf{z}}$$

Example: Calculate the curl of H in Cartesian coordinates due to a current filament along the z axis with current I in the a_z direction?

Solution:



The magnetic field intensity due to a current filament is:

$$H = \frac{I}{2\pi\rho}\mathrm{a}_\emptyset = \frac{I}{2\pi}\left(\frac{-y\mathrm{a}_x + x\mathrm{a}_y}{x^2+y^2}\right)$$

$$Curl\ H = \nabla \times \mathrm{H} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{y}{x^2+y^2} & \frac{x}{x^2+y^2} & 0 \end{vmatrix} = 0$$

Example: A cylindrical conductor of radius 10^{-2} m has an internal magnetic field

$H = 4.77 \times 10^4 (\frac{\rho}{2} - \frac{\rho^2}{3\times10^{-2}})\mathbf{a}_\emptyset$ A/m, what is the total current in the conductor?

Solution:

$$\mathrm{J} = \nabla \times H$$

$$H_\emptyset = 4.77 \times 10^4 \left(\frac{\rho}{2} - \frac{\rho^2}{3\times 10^{-2}}\right) \qquad , H_\rho = 0 \qquad , H_z = 0$$

$$\mathrm{J} = \nabla \times H = \left(\frac{1}{\rho}\frac{\partial H_z}{\partial \emptyset} - \frac{\partial H_\emptyset}{\partial z}\right)\mathbf{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right)\mathbf{a}_\emptyset + \left(\frac{1}{\rho}\frac{\partial(\rho H_\emptyset)}{\partial \rho} - \frac{1}{\rho}\frac{\partial H_\rho}{\partial \emptyset}\right)\mathbf{a}_z$$

$$\mathrm{J} = \nabla \times H = -\frac{\partial H_\emptyset}{\partial z}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial(\rho H_\emptyset)}{\partial \rho}\mathbf{a}_z$$

$$\mathrm{J} = \nabla \times H = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(4.77 \times 10^4\left(\frac{\rho^2}{2} - \frac{\rho^3}{3\times 10^{-2}}\right)\right)\mathbf{a}_z$$

$$\mathrm{J} = \nabla \times H = 4.77 \times 10^4 \frac{1}{\rho}\left(\rho - \frac{\rho^2}{1\times 10^{-2}}\right)\mathbf{a}_z = 4.77 \times 10^4\left(1 - \frac{\rho}{1\times 10^{-2}}\right)\mathbf{a}_z$$

$$I = \int_S \mathrm{J}.dS = \int_0^{2\pi}\int_0^{0.01} 4.77 \times 10^4\left(1 - \frac{\rho}{1\times 10^{-2}}\right).\rho d\rho d\emptyset$$

$$I = 4.77 \times 10^4 * 2\pi \int_0^{0.01}\left(\rho - \frac{\rho^2}{1\times 10^{-2}}\right)d\rho = 5\ A$$

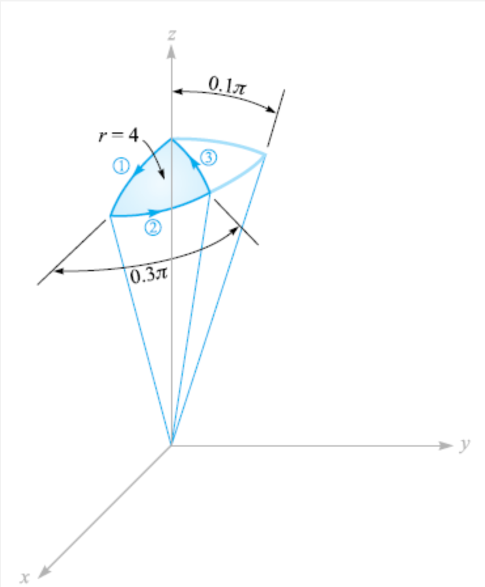
7.5 Stokes' Theorem

Consider an open surface S whose boundary is a closed curve C. ***Stokes' theorem*** states that the integral of the tangential component of a vector field F around C is equal to the integral of the normal component of curl F over S:

$$\oint \mathrm{H}.dL = \int_S (\nabla \times H).dS$$

Example: Consider the portion of a sphere shown in Figure below. The surface is specified by $r = 4$, $0 \le \theta \le 0.1\pi$, $0 \le \emptyset \le 0.3\pi$. If the field $H = 6r\sin\emptyset\, \mathrm{a}_r + 18r\ \sin\theta\cos\emptyset\ \mathrm{a}_\emptyset$, evaluate each side of Stokes' theorem?

Solution:



$$\oint \mathrm{H}.dL = \int_S (\nabla \times H).dS$$

The left side is:

$$\oint \mathrm{H}.dL$$

$$\mathrm{dL} = dr\mathbf{a}_r + rd\theta\mathbf{a}_\theta + r\sin\theta\ d\emptyset\ \mathbf{a}_\emptyset$$

$$\oint \mathrm{H}.dL = \int (6r\sin\emptyset\,\mathrm{a}_r + 18r\ \sin\theta\cos\emptyset\ \mathrm{a}_\emptyset).(dr\mathbf{a}_r + rd\theta\mathbf{a}_\theta + r\sin\theta\ d\emptyset\,\mathbf{a}_\emptyset)$$

$$\oint \mathrm{H}.dL = \int 6r\sin\emptyset\ dr + \int 18r^2\ \sin^2\theta\cos\emptyset\, d\emptyset$$

$$\oint \mathrm{H}.dL = 18r^2\ \sin^2\theta \int_0^{0.3\pi} \cos\emptyset\, d\emptyset = 18(4)^2\ \sin^2(0.1\pi)\sin 0.3\pi = 22.2\ A$$

The right side is:

$$\int_S (\nabla \times H).dS$$

$$\nabla \times H = \frac{1}{r\sin\theta}\left(\frac{\partial(\sin\theta\,\mathrm{H}_\emptyset)}{\partial\theta} - \frac{\partial H_\theta}{\partial\emptyset}\right)\mathbf{a}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\emptyset} - \frac{\partial(rH_\emptyset)}{\partial r}\right)\mathbf{a}_\theta + \frac{1}{r}\left(\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial(H_r)}{\partial\theta}\right)\mathbf{a}_\emptyset$$

$$\nabla \times H = \frac{1}{r\,sin\,\theta}\left(\frac{\partial(18r\ \sin^2\theta\cos\emptyset)}{\partial\theta}\right)\mathbf{a}_{\boldsymbol{r}} + \frac{1}{r}\left(\frac{1}{sin\,\theta}\frac{\partial(6r\sin\emptyset)}{\partial\emptyset} - \frac{\partial(18r^2\ \sin\theta\cos\emptyset)}{\partial r}\right)\mathbf{a}_{\boldsymbol{\theta}} + \frac{1}{r}\left(-\frac{\partial(6r\sin\emptyset)}{\partial\theta}\right)\mathbf{a}_{\emptyset}$$

$$\nabla \times H = \frac{1}{r\,sin\,\theta}(36r\ \sin\theta\cos\theta\cos\emptyset)\mathbf{a}_{\boldsymbol{r}} + \frac{1}{r}\left(\frac{1}{sin\,\theta}6r\cos\emptyset - 36r\ \sin\theta\cos\emptyset\right)\mathbf{a}_{\boldsymbol{\theta}}$$

$$dS = r^2\sin\theta\,d\theta\,d\emptyset\,\mathbf{a}_{\boldsymbol{r}}$$

$$\int_S (\nabla\times H).dS =$$

$$\int_S \left(\frac{1}{r\,sin\,\theta}(36r\ \sin\theta\cos\theta\cos\emptyset)\mathbf{a}_{\boldsymbol{r}} + \frac{1}{r}\left(\frac{1}{sin\,\theta}6r\cos\emptyset - 36r\ \sin\theta\cos\emptyset\right)\mathbf{a}_{\boldsymbol{\theta}}\right).r^2\sin\theta\,d\theta\,d\emptyset\,\mathbf{a}_{\boldsymbol{r}}$$

$$\int_S (\nabla\times H).dS = \int_0^{0.3\pi}\int_0^{0.1\pi}\frac{r^2\sin\theta}{r\,sin\,\theta}(36r\ \sin\theta\cos\theta\cos\emptyset)\,d\theta\,d\emptyset$$

$$\int_S (\nabla\times H).dS = 36r^2\int_0^{0.3\pi}\int_0^{0.1\pi}\sin\theta\cos\theta\cos\emptyset\,d\theta\,d\emptyset$$

$$\int_S (\nabla\times H).dS = 36(4)^2\left[\frac{\sin^2\theta}{2}\right]_0^{0.1\pi}[\sin\emptyset]_0^{0.3\pi}$$

$$\int_S (\nabla\times H).dS = 22.2\ A$$

The left side =the right side

7.6 Magnetic Flux and Magnetic Flux Density الفيض المغناطيسي

Like D, the magnetic field strength H depends only on (moving) charges and is independent of the medium. The force field associated with H is the magnetic flux density B, which is given by

$$B = \mu H$$

Where permeability $\mu = \mu_o \mu_r$

$$\mu_o = 4\pi \times 10^{-7} \quad H/m$$

The relative permeability of the medium, is a pure number very near to unity, except for a small group of ferromagnetic materials

Where B is measured in weber per square meter (Wb/m^2),or in a newer unit adopted in the International System of Units, tesla (T). An older unit that is often used for magnetic flux density is the gauss (G), here 1 T or 1Wb/m2 is the same as 10, 000 G

Let us represent magnetic flux by Φ and define Φ *as* the flux passing through any designated area,

$$\Phi = \int_S B \,.\, dS \quad Wb$$

The sign on Φ may be positive or negative depending upon the choice of the surface normal in dS. The unit of magnetic flux is the weber, Wb.

The magnetic flux lines are closed and do not terminate on a “magnetic charge.” For this reason Gauss’s law for the magnetic field is

$$\oint B \,.\, dS = 0$$

and application of the divergence theorem shows us that

$$\nabla . B = 0$$

Equation (above) is the last of Maxwell’s four equations as they apply to static electric fields and steady magnetic fields

Collecting these equations, we then have for static electric fields and steady magnetic fields:

معادلات ماكسويل للمجالات الكهربائية والمغناطيسية الثابتة

$$\nabla . D = \rho_v$$

$$\nabla \times E = 0$$

$$\nabla \times H = \mathrm{J}$$

$$\nabla . B = 0$$

The corresponding set of four ***integral equations*** that apply to static electric fields and steady magnetic fields is

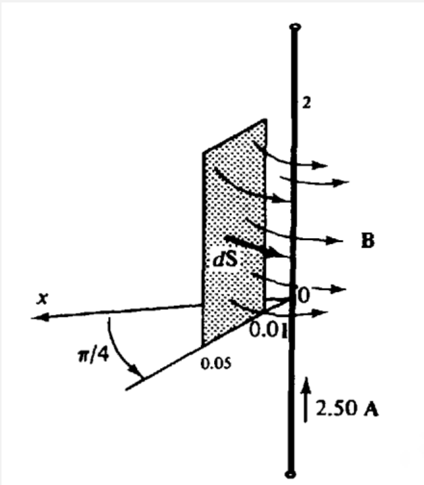
$$\oint D_S \ . dS = \int_{vol} \rho_v dv$$

$$\oint E . dL = 0$$

$$\oint \mathrm{H}. dL = \mathrm{I} \ = \int \mathrm{J} . dS$$

$$\oint B . dS = 0$$

Example: Find the flux crossing the portion of the plane shown in figure below defined by **0.01 < *ρ*< 0.05** m and **0 < z < 2** m. A current filament of **2.5** A along the z axis is in the $\mathbf{a}_z$ direction?



Solution:

$$\Phi = \int_S B . dS$$

$$B = \mu H$$

$$H = \frac{I}{2\pi\rho}\mathbf{a}_\emptyset \qquad , \qquad B = \frac{\mu I}{2\pi\rho}\mathbf{a}_\emptyset$$

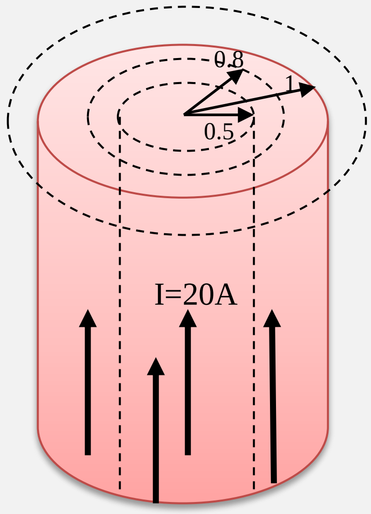
$$dS = d\rho \ dz\mathbf{a}_\emptyset$$

$$\Phi = \int_S \frac{\mu I}{2\pi\rho}\mathbf{a}_\emptyset . d\rho \ dz\mathbf{a}_\emptyset$$

$$\Phi = \int_0^2 \int_{0.01}^{0.05} \frac{\mu I}{2\pi\rho} d\rho \ dz \quad = \frac{2\mu I}{2\pi}[\ln \rho]_{0.01}^{0.05} = \frac{\mu I}{\pi}\ln\frac{0.05}{0.01} = 1.61 \ \mu Wb$$

Example: A solid conductor of circular cross section. If the radius a = 1 mm, the conductor axis lies on the z axis, and the total current in the $\mathbf{a_z}$ direction is 20 A, find: (a) $H_\emptyset$ at ρ= 0.5mm; (b) $B_\emptyset$ at ρ = 0.8 mm; (c) the total magnetic flux per unit length inside the conductor; (d) the total flux for ρ < 0.5 mm; (e) the total magnetic flux outside the conductor.?

Solution:



$(\boldsymbol{a})\ \ H\ at\ \rho = 0.5\ mm$

$$\oint \mathrm{H}.dL = I_{enc}$$

$$I_{enc} = I\frac{\pi\rho^2}{\pi a^2} = 20 * \frac{(0.5)^2}{1^2} = 5\ A$$

$$\oint \mathrm{H}.dL = \int_0^{2\pi} \mathrm{H}_\emptyset\ \rho d\emptyset\ = \mathrm{H}_\emptyset\ \rho\int_0^{2\pi} d\emptyset = 2\pi\rho\ \mathrm{H}_\emptyset$$

$$2\pi\rho\ \mathrm{H}_\emptyset\ = 5$$

$$\mathrm{H}_\emptyset\ = \frac{5}{2\pi\ 0.5 \times 10^{-3}} = 1592\ A/m$$

$(\boldsymbol{b})\ B\ at\ \rho = 0.8\ mm$

$B = \mu H$

$H\ at\ \rho = 0.8\ mm$

$$I_{enc} = I\frac{\pi\rho^2}{\pi a^2} = 20 * \frac{(0.8)^2}{1^2} = 12.8\ A$$

$$\mathrm{H}_\emptyset\ = \frac{12.8}{2\pi\ 0.8 \times 10^{-3}}$$

$$B = \mu\frac{12.8}{2\pi\ 0.8 \times 10^{-3}} = 3.2\ mT$$

$(\boldsymbol{c})$

$$\Phi = \int_S\ B\,.\,dS$$

$B\ inside\ \ = \mu\ H\ inside$

$H\ inside$

$$I_{enc} = I\frac{\rho^2}{a^2}$$

$$\mathrm{H}\ = \frac{I\rho}{2\pi a^2}\mathrm{a}_\emptyset$$

$\therefore B = \dfrac{\mu I \rho}{2\pi a^2} \mathrm{a}_{\emptyset}$

$dS = d\rho\, dz \mathrm{a}_{\emptyset}$

$\Phi = \int_S B \,.\, dS = \int_0^L \int_0^{0.001} \dfrac{\mu I \rho}{2\pi a^2} \mathrm{a}_{\emptyset} .\, d\rho\, dz \mathrm{a}_{\emptyset}$

$= \dfrac{\mu I}{2\pi a^2} * L \left[\dfrac{\rho^2}{2}\right]_0^{0.001}$

$= \dfrac{\mu * 20}{2\pi (0.001)^2} * L \left[\dfrac{(0.001)^2}{2}\right] = 2L\, \mu Wb$

$\dfrac{\Phi}{L} = 2L\, \mu Wb/m$

($\boldsymbol{d}$) Total flux for $\rho < 0.5$ mm

$\Phi = \int_S B \,.\, dS = \int_0^L \int_0^{0.005} \dfrac{\mu I \rho}{2\pi a^2} \mathrm{a}_{\emptyset} .\, d\rho\, dz \mathrm{a}_{\emptyset}$

$= \dfrac{\mu * 20}{2\pi (0.001)^2} * L \left[\dfrac{(0.005)^2}{2}\right] = 0.5L\, \mu Wb$

($\boldsymbol{e}$) the total magnetic flux outside the conductor

$I_{enc} = I$

$\mathrm{H} = \dfrac{I}{2\pi\rho} \mathrm{a}_{\emptyset} \qquad , \therefore B = \dfrac{\mu I}{2\pi\rho} \mathrm{a}_{\emptyset}$

$\Phi = \int_S B \,.\, dS = \int_0^L \int_1^{\infty} \dfrac{\mu I}{2\pi\rho} \mathrm{a}_{\emptyset} .\, d\rho\, dz \mathrm{a}_{\emptyset}$

$= \dfrac{\mu I}{2\pi} * L [\ln \rho]_0^{\infty} = \dfrac{\mu I}{2\pi} * L\, [\ln \infty - \ln 1]$

$\dfrac{\Phi}{L} = \infty$

7.7 The Vector Magnetic Potentials (A) الجهد المغناطيسي المتجه

This vector field is one which is extremely useful in studying radiation from antennas as well as radiation leakage from transmission lines, waveguides, and microwave ovens. The vector magnetic potential may be used in regions where the current density is zero or nonzero.
Our choice of a vector magnetic potential is indicated by noting that

$$\nabla . B = 0$$

The divergence of the curl of any vector field is zero. Therefore, we select

$$B = \nabla \times A$$

Where A signifies a ***vector magnetic potential***, and we automatically satisfy the condition that the magnetic flux density shall have zero divergence, the unit of A is Wb/m. The **H** field is

$$H = \frac{1}{\mu} \nabla \times A$$

The vector magnetic potential A can be determined from the known currents in the region of interest. For the three standard current configurations the expressions are as follows.

- ***Current filament***

$$A = \oint \frac{\mu I \, dL}{4\pi R}$$

- ***Sheet current:***

$$A = \int_S \frac{\mu K \, dS}{4\pi R}$$

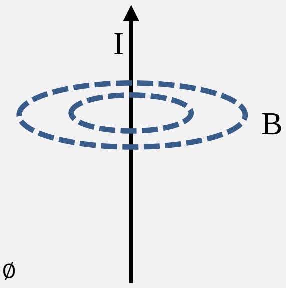
- ***Volume current:***

$$A = \int_V \frac{\mu \mathrm{J} \, dv}{4\pi R}$$

Here, R is the distance from the current element to the point at which the vector magnetic potential is being calculated.

Example: Obtain the vector magnetic potential A in the region surrounding an infinitely long, straight, filamentary current?

Solution:



$$H = \frac{I}{2\pi\rho}\mathrm{a}_\phi \qquad ,\therefore B = \frac{\mu I}{2\pi\rho}\mathrm{a}_\phi$$

$$\nabla \times A = B$$

$$\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\mathbf{a}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\mathbf{a}_\phi + \left(\frac{1}{\rho}\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{1}{\rho}\frac{\partial A_\rho}{\partial \phi}\right)\mathbf{a}_z = \frac{\mu I}{2\pi\rho}\mathrm{a}_\phi$$

$$\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\mathbf{a}_\phi = \frac{\mu I}{2\pi\rho}\mathrm{a}_\phi$$

$$-\frac{\partial A_z}{\partial \rho} = \frac{\mu I}{2\pi\rho}$$

$$A_z = \int \frac{\mu I}{2\pi\rho}\, d\rho = \frac{\mu I}{2\pi}\ln\rho + c$$

$$let\ A_z = 0\ at\ \rho = \rho_0$$

$$A_z = \frac{\mu I}{2\pi}\ln\frac{\rho_0}{\rho}$$

Example: Let the vector magnetic potential **A** = (3*y* − *z*)**a***x* + 2*xz***a***y* Wb/m in a certain region of free space.(*a*) Show that ∇ · **A** = 0. (*b*) At *P*(2,−1, 3), find **A**, **B**, **H**, and **J**?

Solution:

$$\nabla . A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \frac{\partial}{\partial x}(3y - z) + \frac{\partial}{\partial y}(2xz) = 0$$

$$A\ at\ the\ point\ P = (3*(-1) - 3)\mathrm{a}_x + 2*2*3\mathrm{a}_y = -6\mathrm{a}_x + 12\mathrm{a}_y$$

$$B = \nabla \times A = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (3y - z) & (2xz) & 0 \end{vmatrix}$$

$$= \frac{-\partial}{\partial z}(2xz)\mathbf{a}_x + \frac{\partial}{\partial z}(3y - z)\mathbf{a}_y + \left[\frac{\partial}{\partial x}(2xz) - \frac{\partial}{\partial y}(3y - z)\right]\mathbf{a}_z$$

$$B = -2x\mathrm{a}_x - \mathrm{a}_y + (2z - 3)\mathrm{a}_z$$

B at the point P is:

$$B = -4\mathrm{a}_x - \mathrm{a}_y + 3\mathrm{a}_z$$

$$H = \frac{B}{\mu} = \frac{-2x\mathrm{a}_x - \mathrm{a}_y + (2z - 3)\mathrm{a}_z}{\mu}$$

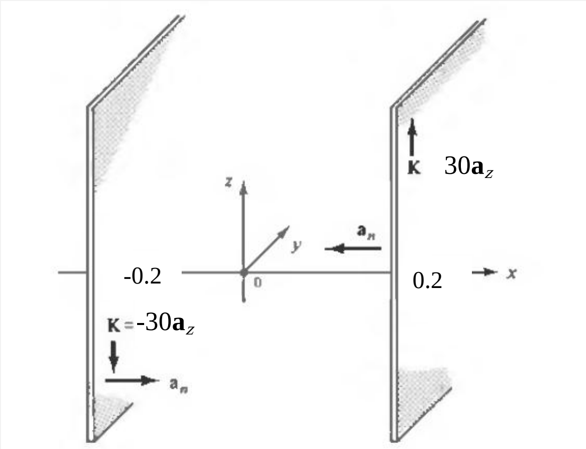
$$J = \nabla \times H = \frac{1}{\mu}\nabla \times B = \frac{1}{\mu}\begin{vmatrix} \mathbf{a_x} & \mathbf{a_y} & \mathbf{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -2x & -1 & (2z-3) \end{vmatrix} =$$

$$= \left[\frac{\partial}{\partial y}(2z-3) - \frac{\partial}{\partial z}(-1)\right]\mathbf{a_x} + \left[\frac{-\partial}{\partial x}(2z-3) + \frac{\partial}{\partial z}(1)\right]\mathbf{a_y} + \left[\frac{\partial}{\partial x}(-1) - \frac{\partial}{\partial y}(-2\mathrm{x})\right]\mathbf{a_z} = 0$$

Example: Planar current sheets of **K** = 30$\mathbf{a}_z$ A/m and −30$\mathbf{a}_z$ A/m are located in free space at x = 0.2 and x = −0.2, respectively. For the region −0.2 < x < 0.2 (a) find **H**; (b) find **B**; (c) obtain an expression for **A** if **A** = 0 at (0.1, 0.5, 0.4)?

Solution:

$for\ -5 < x < 5$



$$H_1 = \frac{1}{2}K_1 \times \mathrm{a}_N$$

$$H_1 = \frac{1}{2}30\mathrm{a}_z \times -\mathrm{a}_x = -15\mathrm{a}_y$$

$$H_2 = \frac{1}{2} - 30\mathrm{a}_z \times \mathrm{a}_x = -15\mathrm{a}_y$$

$$H = H_1 + H_2 = -30\mathrm{a}_y$$

$$B = \mu H = -30\mu\mathrm{a}_y = -120\pi\mathrm{a}_y$$

$$\nabla \times A = B$$

$$\begin{vmatrix} \mathbf{a_x} & \mathbf{a_y} & \mathbf{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = -120\pi\mathrm{a}_y$$

$$\left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\mathbf{a_y} = -120\pi\mathrm{a}_y$$

$$-\frac{\partial A_z}{\partial x} = -120\pi$$

$$A_z = 120\pi x + C$$

$$0 = 120\pi(0.1) + C \quad ,\therefore C = -12\pi$$

$$A_z = 12\pi(10x - 1)$$

Example: Assume that $A = 50\rho^2 \mathrm{a}_z$ Wb/m in a certain region of free space. (a) Find **H** and **B**. (b) Find **J**. (c) Use **J** to find the total current crossing the surface $0 \leq \rho \leq 1$, $0 \leq \emptyset \leq 2\pi$ and $z = 0$?

Solution:

$$(\boldsymbol{a})\ \ B = \nabla \times A = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \emptyset} - \frac{\partial A_\emptyset}{\partial z}\right)\mathbf{a}_{\boldsymbol{\rho}} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\mathbf{a}_\emptyset + \left(\frac{1}{\rho}\frac{\partial(\rho A_\emptyset)}{\partial \rho} - \frac{1}{\rho}\frac{\partial A_\rho}{\partial \emptyset}\right)\mathbf{a}_{\boldsymbol{z}}$$

$$B = \frac{1}{\rho}\frac{\partial A_z}{\partial \emptyset}\mathbf{a}_{\boldsymbol{\rho}} - \frac{\partial A_z}{\partial \rho}\mathbf{a}_\emptyset = -\frac{\partial}{\partial \rho}(50\rho^2)\mathbf{a}_\emptyset = -100\rho\mathbf{a}_\emptyset$$

$$H = \frac{B}{\mu} = \frac{-100\rho\mathbf{a}_\emptyset}{\mu}$$

$$(\boldsymbol{b})\ \ \mathrm{J} = \nabla \times H = \left(\frac{1}{\rho}\frac{\partial H_z}{\partial \emptyset} - \frac{\partial H_\emptyset}{\partial z}\right)\mathbf{a}_{\boldsymbol{\rho}} + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right)\mathbf{a}_\emptyset + \left(\frac{1}{\rho}\frac{\partial(\rho H_\emptyset)}{\partial \rho} - \frac{1}{\rho}\frac{\partial H_\rho}{\partial \emptyset}\right)\mathbf{a}_{\boldsymbol{z}}$$

$$\mathrm{J} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho H_\emptyset)\mathbf{a}_{\boldsymbol{z}} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{-100\rho}{\mu}\right)\mathbf{a}_{\boldsymbol{z}} = \frac{-200}{\mu}\mathbf{a}_{\boldsymbol{z}} \quad A/m^2$$

$(\boldsymbol{c})$

$$I = \int \mathrm{J}.dS = \int_0^{2\pi}\int_0^1 \frac{-200}{\mu}\ \rho d\rho d\emptyset = \frac{-200\pi}{\mu}$$

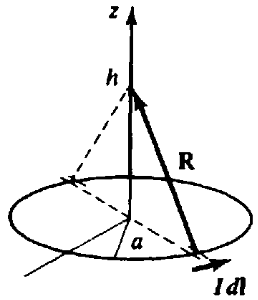
.

Homework

Q₁. *Evaluate both sides of Stokes' theorem for the field* $\boldsymbol{H} = 6xy\boldsymbol{a}_x - 3y^2\boldsymbol{a}_y$ *A/m and the rectangular path around the region,* $2 \le x \le 5$, $-1 \le y \le 1$, $z = 0$.? ***Ans:****-126*

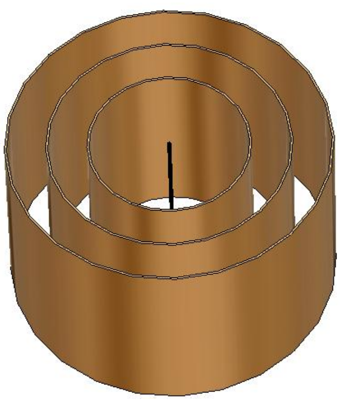
Q₂: *Given* $A = -\rho^2/4\ \mathrm{a}_z$ *wb/m. Find the total magnetic flux crossing the surface* $\emptyset = \pi/2$, $1 \le \rho \le 2$ *and* $0 \le z \le 5$. ***Ans:*** *3.75wb*

Q₃: *Find H on the axis of a circular current loop of radius a. Specialize the result to the center of the loop.*



Ans: at h=0, $H = \frac{I}{2a}\mathrm{a}_z$

Q₄: *An* ***infinite filament*** *on the z axis carries 20π mA in the* $\mathbf{a}_z$ *direction.* ***Three uniform cylindrical current sheets*** *are also present: 400 mA/m at ρ = 1 cm, −250 mA/m at ρ = 2 cm, and −300 mA/m at ρ = 3 cm. Calculate* $H_\emptyset$ *at ρ = 0.5, 1.5, 2.5, and 3.5 cm*



Ans: 2 A/m, 933 mA/m, 360 mA/m, 0

Q₅: ***A current filament*** *on the z axis carries a current of 7 mA in the* $\boldsymbol{a}_z$ *direction,* ***and current sheets*** *of* $\mathbf{0.5\ a_z}$ *A/m and* $\mathbf{-0.2\ a_z}$ *A/m are located at* **ρ= 1** *cm and* **ρ = 0.5** *cm, respectively. Calculate* ***H*** *at: (a) ρ = 0.5 cm; (b) ρ = 1.5 cm; (c) ρ = 4 cm. (d) What current sheet should be located at ρ = 4 cm so that* ***H*** *= 0 for all ρ > 4 cm?*

Ans: $2.3 \times 10^{-2}\ \mathbf{a}_\emptyset$, $0.34\ \mathbf{a}_\emptyset$, $0.13\ \mathbf{a}_\emptyset$